

Mathematica 11.3 Integration Test Results

Test results for the 21 problems in "4.2.8 (a+b cos)^m (c+d trig)^n.m"

Problem 9: Result more than twice size of optimal antiderivative.

$$\int \frac{(c + d \operatorname{Sec}[e + f x])^4}{a + b \operatorname{Cos}[e + f x]} dx$$

Optimal (type 3, 247 leaves, 12 steps):

$$\frac{2 (a c - b d)^4 \operatorname{ArcTan}\left[\frac{\sqrt{a-b} \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]}{\sqrt{a+b}}\right]}{a^4 \sqrt{a-b} \sqrt{a+b} f} + \frac{d^3 (4 a c - b d) \operatorname{ArcTanh}[\operatorname{Sin}[e + f x]]}{2 a^2 f} +$$

$$\frac{d (2 a c - b d) (2 a^2 c^2 - 2 a b c d + b^2 d^2) \operatorname{ArcTanh}[\operatorname{Sin}[e + f x]]}{a^4 f} + \frac{d^4 \operatorname{Tan}[e + f x]}{a f} +$$

$$\frac{d^2 (6 a^2 c^2 - 4 a b c d + b^2 d^2) \operatorname{Tan}[e + f x]}{a^3 f} + \frac{d^3 (4 a c - b d) \operatorname{Sec}[e + f x] \operatorname{Tan}[e + f x]}{2 a^2 f} + \frac{d^4 \operatorname{Tan}[e + f x]^3}{3 a f}$$

Result (type 3, 952 leaves):

$$\begin{aligned}
 & \frac{2 (a c - b d)^4 \operatorname{ArcTanh}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]}{\sqrt{-a^2+b^2}}\right] \operatorname{Cos}[e+f x]^4 (c+d \operatorname{Sec}[e+f x])^4}{a^4 \sqrt{-a^2+b^2} f (d+c \operatorname{Cos}[e+f x])^4} + \\
 & \left((-8 a^3 c^3 d + 12 a^2 b c^2 d^2 - 4 a^3 c d^3 - 8 a b^2 c d^3 + a^2 b d^4 + 2 b^3 d^4) \operatorname{Cos}[e+f x]^4 \right. \\
 & \quad \left. \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right] (c+d \operatorname{Sec}[e+f x])^4 \right) / \left(2 a^4 f (d+c \operatorname{Cos}[e+f x])^4 \right) + \\
 & \left((8 a^3 c^3 d - 12 a^2 b c^2 d^2 + 4 a^3 c d^3 + 8 a b^2 c d^3 - a^2 b d^4 - 2 b^3 d^4) \operatorname{Cos}[e+f x]^4 \right. \\
 & \quad \left. \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right] (c+d \operatorname{Sec}[e+f x])^4 \right) / \left(2 a^4 f (d+c \operatorname{Cos}[e+f x])^4 \right) + \\
 & \frac{(12 a c d^3 + a d^4 - 3 b d^4) \operatorname{Cos}[e+f x]^4 (c+d \operatorname{Sec}[e+f x])^4}{12 a^2 f (d+c \operatorname{Cos}[e+f x])^4 \left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right)^2} + \\
 & \frac{d^4 \operatorname{Cos}[e+f x]^4 (c+d \operatorname{Sec}[e+f x])^4 \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]}{6 a f (d+c \operatorname{Cos}[e+f x])^4 \left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right)^3} + \\
 & \frac{d^4 \operatorname{Cos}[e+f x]^4 (c+d \operatorname{Sec}[e+f x])^4 \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]}{6 a f (d+c \operatorname{Cos}[e+f x])^4 \left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right)^3} + \\
 & \frac{(-12 a c d^3 - a d^4 + 3 b d^4) \operatorname{Cos}[e+f x]^4 (c+d \operatorname{Sec}[e+f x])^4}{12 a^2 f (d+c \operatorname{Cos}[e+f x])^4 \left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right)^2} + \\
 & \left(\operatorname{Cos}[e+f x]^4 (c+d \operatorname{Sec}[e+f x])^4 \left(18 a^2 c^2 d^2 \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right] - \right. \right. \\
 & \quad \left. \left. 12 a b c d^3 \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right] + 2 a^2 d^4 \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right] + 3 b^2 d^4 \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right] \right) \right) / \\
 & \left(3 a^3 f (d+c \operatorname{Cos}[e+f x])^4 \left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right) \right) + \\
 & \left(\operatorname{Cos}[e+f x]^4 (c+d \operatorname{Sec}[e+f x])^4 \left(18 a^2 c^2 d^2 \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right] - \right. \right. \\
 & \quad \left. \left. 12 a b c d^3 \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right] + 2 a^2 d^4 \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right] + 3 b^2 d^4 \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right] \right) \right) / \\
 & \left(3 a^3 f (d+c \operatorname{Cos}[e+f x])^4 \left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right) \right)
 \end{aligned}$$

Problem 16: Unable to integrate problem.

$$\int \frac{\sqrt{c+d \operatorname{Sec}[e+f x]}}{a+b \operatorname{Cos}[e+f x]} dx$$

Optimal (type 4, 213 leaves, 4 steps):

$$\frac{2 \sqrt{c+d} \operatorname{Cot}[e+f x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{c+d \operatorname{Sec}[e+f x]}}{\sqrt{c+d}}\right], \frac{c+d}{c-d}\right] \sqrt{\frac{d(1-\operatorname{Sec}[e+f x])}{c+d}} \sqrt{\frac{-d(1+\operatorname{Sec}[e+f x])}{c-d}}}{a f} +$$

$$\frac{2(a c-b d) \operatorname{EllipticPi}\left[\frac{2 a}{a+b}, \operatorname{ArcSin}\left[\frac{\sqrt{1-\operatorname{Sec}[e+f x]}}{\sqrt{2}}\right], \frac{2 d}{c+d}\right] \sqrt{\frac{c+d \operatorname{Sec}[e+f x]}{c+d}} \operatorname{Tan}[e+f x]}{a(a+b) f \sqrt{c+d \operatorname{Sec}[e+f x]} \sqrt{-\operatorname{Tan}[e+f x]^2}}$$

Result (type 8, 29 leaves):

$$\int \frac{\sqrt{c+d \operatorname{Sec}[e+f x]}}{a+b \operatorname{Cos}[e+f x]} dx$$

Problem 17: Unable to integrate problem.

$$\int \frac{1}{(a+b \operatorname{Cos}[e+f x]) \sqrt{c+d \operatorname{Sec}[e+f x]}} dx$$

Optimal (type 4, 102 leaves, 2 steps):

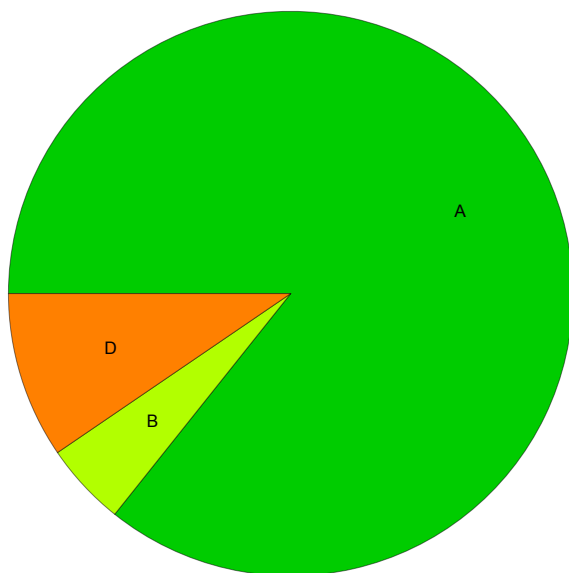
$$\frac{2 \operatorname{EllipticPi}\left[\frac{2 a}{a+b}, \operatorname{ArcSin}\left[\frac{\sqrt{1-\operatorname{Sec}[e+f x]}}{\sqrt{2}}\right], \frac{2 d}{c+d}\right] \sqrt{\frac{c+d \operatorname{Sec}[e+f x]}{c+d}} \operatorname{Tan}[e+f x]}{(a+b) f \sqrt{c+d \operatorname{Sec}[e+f x]} \sqrt{-\operatorname{Tan}[e+f x]^2}}$$

Result (type 8, 29 leaves):

$$\int \frac{1}{(a+b \operatorname{Cos}[e+f x]) \sqrt{c+d \operatorname{Sec}[e+f x]}} dx$$

Summary of Integration Test Results

21 integration problems



A - 18 optimal antiderivatives

B - 1 more than twice size of optimal antiderivatives

C - 0 unnecessarily complex antiderivatives

D - 2 unable to integrate problems

E - 0 integration timeouts